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Effect of frequency modulated signal on chaotic motion in brusselator chemical reaction

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Abstract
The various dynamical behaviours of the Brusselator chemical reaction and the effect of frequency modulated signal acted on the reaction as a chaotic oscillator has been investigated. We considered both narrow band frequency modulated (NBFM) and wide band frequency modulated (WBFM) signals to study the dynamics of the system. A variety of features such as different routes to chaos, periodic windows, reverse period-doubling bifurcation, chaotic orbit, and periodic-bubbles are found to occur due to the applied signal. The numerical results are substantiated by bifurcation diagram, phase portrait and Poincare map.

Keywords: brusselator chemical reaction, frequency modulated signal, periodic bubbles, reverse periodic orbit, chaos

1. Introduction

Nonlinear chemical systems are, in general, prone to exhibit an extraordinarily rich spectrum of dynamical behaviours [1-5]. Recent studies have shown that the influence of different kinds of additional perturbations and external forcing [6-10] on these systems is considerable as they alter the dynamical behaviours drastically. The Brusselator is one of the simplest model in nonlinear chemical system. This model was first proposed by Prigogine *et al.* [11]. It is characterized by the equation,

$$\begin{aligned} A &\xrightarrow{K_1} X \\ B+X &\xrightarrow{K_2} Y+D \\ 2X+Y &\xrightarrow{K_3} 3X \\ X &\xrightarrow{K_4} E. \end{aligned} \qquad (1)$$

Where *A* and *B* are constant components, *D* and *E* are products and *X* and *Y* are the two components variable in time and space. In recent years many researchers studied the dynamics of Eq. (1) under the excitation of external periodic forces [12-21]. The dynamics of the Brusselator chemical reaction can be described by a system of two ordinary differential equations. In our present work, we numerically analyse the effect of NBFM and WBFM signal on chaotic motion in the Brusselator chemical system. In dimensionless form, the equation of motion of the Brusselator chemical reaction with NBFM signal is given by

$$\begin{aligned} \dot{x} &= a-(b+1)x+x^2y+F(\cos\omega t - G\sin\Omega t\sin\omega t) \\ \dot{y} &= bx-x^2y \end{aligned} \qquad (2)$$

and with WBFM signal is given by

$$\begin{aligned} \dot{x} &= a-(b+1)x+x^2y+F\sin(\omega t+G\cos\Omega t) \\ \dot{y} &= bx-x^2y, \end{aligned} \qquad (3)$$

where *a* and *b* are constants with $a, b > 0$, *x* and *y* represent the dimensionless concentration of two reactants, *F* is the unmodulated carrier amplitude, *G* is the modulation index, ω and Ω are the two frequencies of the external signals. Recently, the frequency modulated signal has been applied to certain nonlinear system [22-25] to investigate certain nonlinear phenomena. The main objective of this paper is to study numerically the effect of both NBFM and WBFM signals on chaotic motion in the system (Eqs. (2) and (3)).

The paper is organized as follows. In section 2, we study the effect of NBFM signal on chaotic motion in Eq.(2). The external force has two frequencies ω and Ω. First we study the case ω = Ω and then ω ≠ Ω. In section 3 we analyze the effect of WBFM signal on chaotic motion in Eq.(3) for the cases ω = Ω and also ω ≠ Ω. Finally section 4 contains conclusion of the present work.

2. Effect of NBFM Signal on Chaos in the Brussellator System

In this section, we numerically study the effect of NBFM signal on chaotic motion in the Brusselator system (Eq. (2)) with the frequencies ω = Ω and then ω ≠ Ω. For our numerical study we fix the parameters values as *a* = 0.4, *b* = 1.2, ω = 0.81, Ω = 0.81 and (√5 + 1) / 2. Eqs.(2) and (3) are solved by fourth-order Runge-Kutta method with time step size 2π / ω. Numerical solution corresponding to first 500 drive cycle is left as transient. We analyze the response of the system (Eqs.(2) and (3)) by varying the amplitudes *F* and *G* of the signal for the cases (i) *G* = 0 and *F* is varied, (ii) *G* is fixed and *F* is varied and (iii) *F* is fixed and *G* is varied.

2.1 Effect of NBFM signal in Eq. (2) with ω = Ω

First we study the effect of NBFM signal with the frequencies of ω = Ω = 0.81. Fig.1(a) shows the bifurcation diagram where *F* is set to 0.1 while *G* is varied. As *G* is increased from -2π, a stable period-*T* orbit occurs, which persists up to *G* = -2.00845 and then it loses it stability giving birth to a period-2*T* orbit. System (Eq.(2)) then undergoes further period-doubling bifurcations as the control parameters *G* is smoothly varied. For example, at *G* = 0.32969, the period–2*T* orbit becomes unstable and gives birth to period–4*T* orbit. This cascade of bifurcations continue further as 8*T*, 16*T*,..., orbits and accumulates at $G = G_c$ = 0.09279. At this critical value of *G*, onset of chaotic motion occurs. When the parameter *G* is further increased from G_c one finds that chaotic orbit persists for range of *G* values interspersed by period-3 and period-4 windows, and reverse period–doubling bifurcation. The bifurcation diagram corresponding to *F* = 0.2 and $G \in [-2\pi, 2\pi]$ is shown in Fig. 1(b). As *G* is increased from -2π, the period–*T* orbit persists up to *G* = 1.61525 and then a period–2*T* solution is developed. This is followed by bifurcation to 4*T* solution, 8*T* solution and so on. Onset of chaos takes place at $G = G_c$ = 0.094374. Further increase of *G*, reverse periodic orbits are developed at *G* = 1.880218. For *F* = 0.5, only period–*T* orbit occurs which is shown in Fig.1(c). An example of period-2 attractor and chaotic attractor is shown in Fig.2.

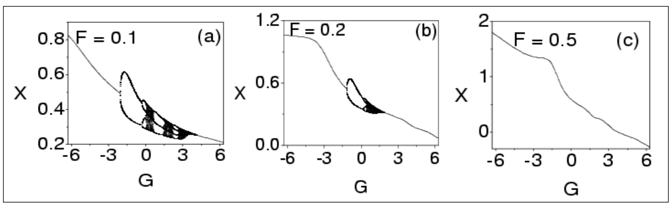

Fig 1: Bifurcation diagrams for the system (Eq. (2)) driven by NBFM signal in (*G*, *X*) plane for three values of *F*. The other parameters values are *a* = 0.4, *b* = 1.2 and ω = Ω = 0.81.

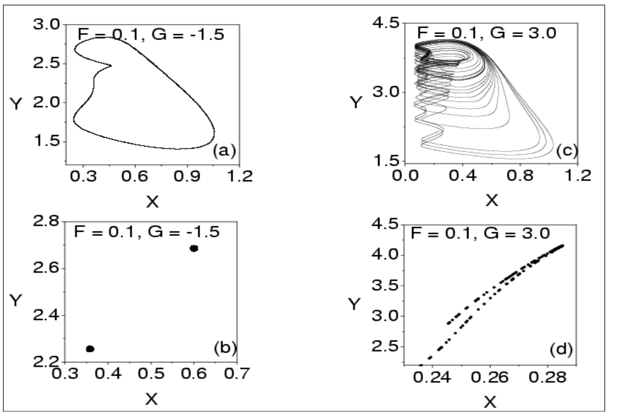

Fig 2: Phase portraits and the corresponding Poincare maps for the system (Eq. (2)) driven by NBFM signal (a-b) period-2 attractor and (c-d) chaotic attractor. The other parameters values are *a* = 0.4, b = 1.2 and ω = Ω = 0.81.

We show the effect of the control parameter *F* by fixing the values of *G*. Fig. 3(a) is the bifurcation diagram obtained by varying *F* from -1 to +1 and *G* is set to zero. As *F* is increased from -1, period-*T* orbit persists up to *F* = -0.5 and then a period-2*T* solution is developed. This is followed by bifurcation to 4*T* solution, 8*T* solution and so on. Onset of chaos takes place at $F = F_c$ = -0.09729. Further increase of *F* from F_c, a reverse period-doubling occurs, namely,…, →8*T*, →4*T* toward a 2*T* solution. The influence of *G*, namely, *G* = 0.1, 1.0 and 5.0 is also studied. The effect of *F* can be clearly seen in the bifurcation diagrams Figs. 3(b), (c) and (d). An example of chaotic attractor and period-3 attractor is shown in Fig.4.

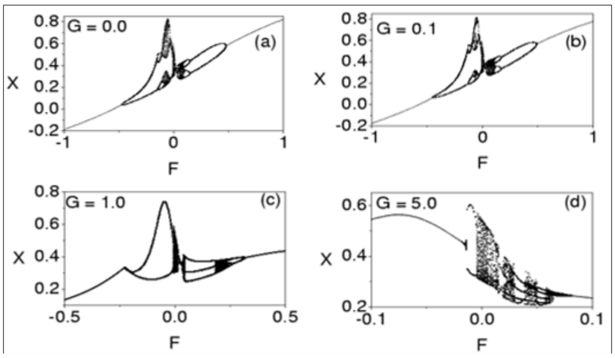

Fig 3: Bifurcation diagrams for the system (Eq.(2)) driven by NBFM signal in (*F*, *X*) plane for four values of *G*. The other parameters values are *a* = 0.4, *b* = 1.2 and ω = Ω = 0.81.

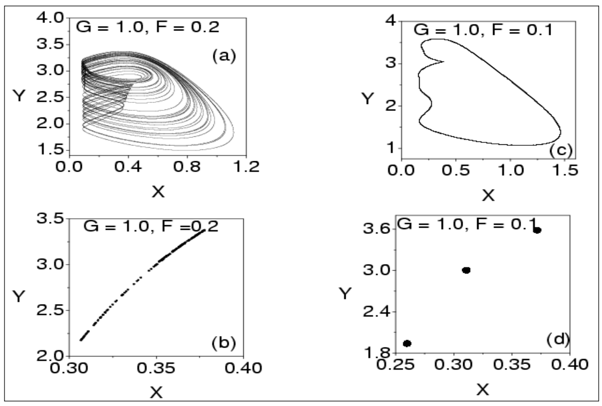

Fig 4: Phase portraits and the corresponding Poincare maps for the system (Eq. (2)) driven by NBFM signal (a-b) chaotic attractor and (c-d) period-3 attractor. The other parameters values are *a* = 0.4, *b* = 1.2 and ω = Ω = 0.81.

2.2 Effect of NBFM signal in Eq. (2) with ω ≠ Ω

Next, we consider the system (Eq.2) subjected to NBFM signal with two incommensurate frequencies. In our numerical study we fixed $\omega = 0.81$ and $\Omega = (\sqrt{5}+1)/2$. The external force is now quasiperiodic. Fig.5 present bifurcation diagrams for three set of values of the parameter *F*. In Fig.5 (a), *F* is kept at 0.1, while *G* is varied. Period–doubling bifurcation is widely observed. For the range - $2\pi < G < 0.0972$, period–doubling orbits exist. At *G* = 0.0972 period–doubling orbits bifurcate into chaotic orbits. On increasing the value of *G* one can again observe chaotic orbits, reverse period–doubling bifurcation and periodic bubbles. The bifurcation patterns for *F* = 0.2 and 0.5 is shown in Figs.5 (b) and (c). In Figs.5(b) and (c), only periodic behaviours is observed. Hence under this kind of perturbation, the chaotic behaviour is completely reduced, leading to a kind of controlling of the complex dynamics in the corresponding parameter region.

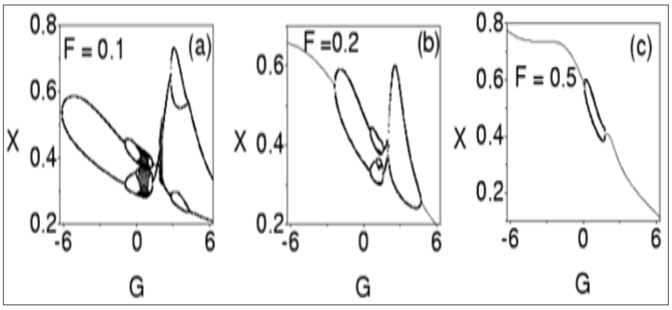

Fig 5: Bifurcation diagrams for the system (Eq.(2)) driven by NBFM signal in (*G, X)* plane for three values of *F*. The other parameters values are *a* = 0.4, b = 1.2, $\omega = 0.81$ and $\Omega = (\sqrt{5}+1)/2$

Now we investigate the dynamics of the system (Eq.2) under the influence of parameter *F* for three values of *G*. The bifurcation diagram corresponding to *G* = 0.1 and $F \in [-1,+1]$ is shown in Fig.6(a). The influence of the control parameter *F* on the dynamics of the two fixed values of *G*, namely, *G* = 1.0 and 5.0 is also studied. The effect of *F* can be clearly seen in the bifurcation diagrams Figs.6 (b) and c. In Fig.6 for certain range of values of the control parameter *F*, period–doubling orbit, reverse period–doubling orbit, chaotic orbit and quasiperiodic orbit are found to occur. An example of quasiperiodic attractor and chaotic attractor is shown in Fig.7.

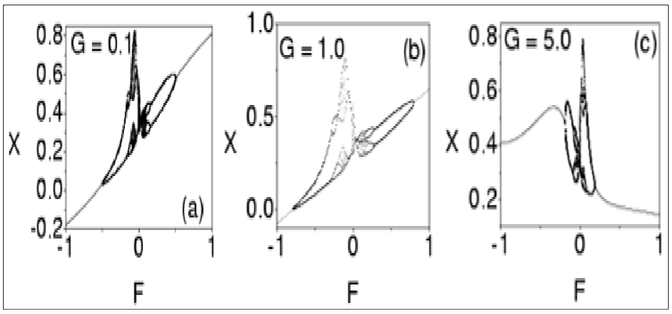

Fig 6: Bifurcation diagrams for the system (Eq. (2)) driven by NBFM signal in (*F*, *X*) plane for three values of *G*. The other parameters values are *a* = 0.4, *b* = 1.2, $\omega = 0.81$ and $\Omega = (\sqrt{5}+1)/2$.

3. Effect of WBFM Signal on Chaos in the Brusselator System

In this section we numerically study the occurrence of bifurcation of periodic orbits leading to chaotic motion and bifurcation of them in the system (Eq.(3)) with WBFM signal for the frequencies ω = Ω and ω ≠ Ω.

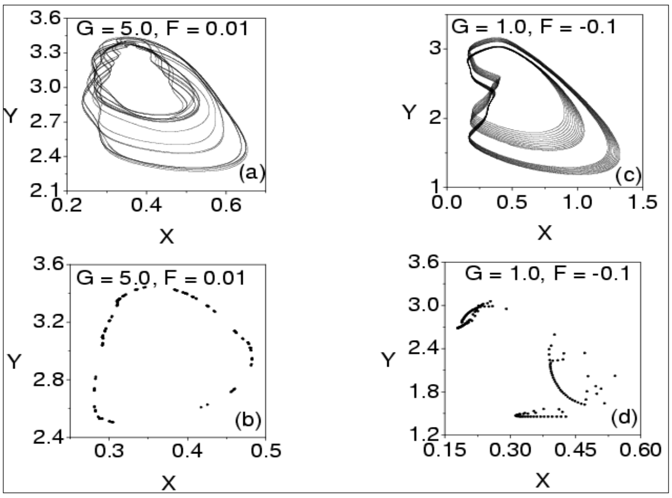

Fig 7: Phase portraits and the corresponding Poincare maps for the system (Eq.(2)) driven by NBFM signal (a-b) quasiperiodic attractor and (c-d) chaotic attractor. The other parameters values are a = 0.4, *b* = 1.2 and $\omega = 0.81$ and $\Omega = (\sqrt{5}+1)/2$

3.1 Effect of WBFM signal with ω = Ω

First we analyze the effect of WBFM signal on chaos in system (Eq.3) for the case ω = Ω. Fig.8 shows the bifurcation diagram of *F* versus *X* for three values of *G* namely *G* = 0.0, 1.0 and 5.0 with $\omega = \Omega = 0.81$. When *F* is varied from small values, the system (Eq.3) with NBFM signal admits variety of bifurcations such as period–doubling, chaos, periodic windows, reverse period–doubling etc. which are clearly seen in Figs.8(a), (b) and (c).

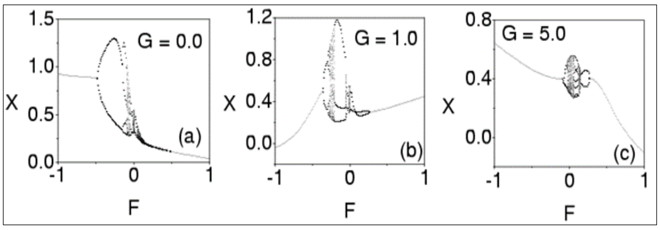

Fig 8: Bifurcation diagrams for the system (Eq.(3)) driven by WBFM signal in (*F*, *X*) plane for three values of *G*. The other parameters values are *a* = 0.4, b = 1.2 and ω = Ω = 0.81.

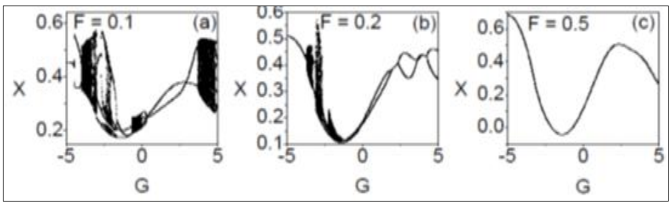

Fig 9: Bifurcation diagrams for the system (Eq.(3)) driven by WBFM signal in (*G*, *X*) plane for three values of *F*. The other parameters values are *a* = 0.4, b = 1.2 and ω = Ω = 0.81.

Now we show the effect of the control parameter *G* by fixing the values of *F*. Fig.9 is the bifurcation diagrams obtained by varying *G* from -5 to +5 for three values of *F*. In Figs.9(a), and 9(b), we observed various bifurcations such as period–doubling, chaos, reverse period-doubling and periodic bubbles. But in Fig. 9(c), the system (Eq.3) shows only periodic behaviour.

3.2 Effect of WBFM signal with ω ≠ Ω

In this section, we analyse the effect of WBFM signal for the case ω ≠ Ω. Fig.10 present bifurcation diagrams for three values of G. In our numerical study we fixed $\omega=1,\ \Omega=(\sqrt{5}+1)/2$, G = 0.1, 1.0 and 5.0 and $F\in[-1,+1]$. The bifurcation diagrams are almost identical corresponding to the values of G = 0.1, 1.0 and 5.0. This is evident from the Figs.10 (a), (b) and (c). But in contrast, when G is varied from -5 to +5, for three values of F such as F = 0.1, 0.2, 0.5, the bifurcation patterns are completely different, which are clearly evident in Figs.11(a), (b) and (c). Here again we observed period–doubling bifurcation, chaotic orbits, reverse period–doubling bifurcation and quasiperiodic orbit.

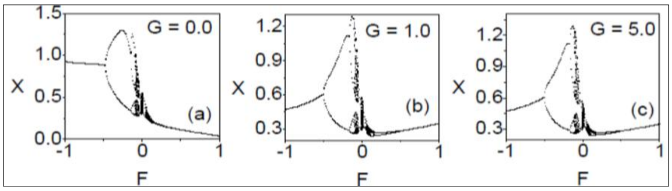

Fig 11: Bifurcation diagrams for the system (Eq.(3) driven by WBFM signal in (F,X) plane for three values of G. The other parameters values are a = 0.4, b = 1.2, ω = 0.81 and Ω = (√5 + 1) / 2.

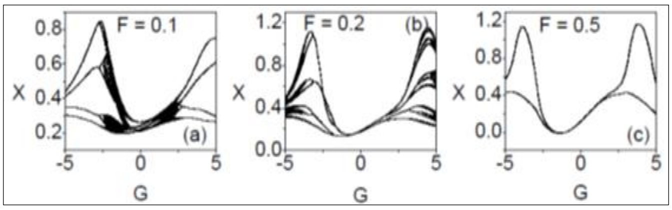

Fig 11: Bifurcation diagrams for the system (Eq.(3)) driven by WBFM signal in (G, X) plane for three values of F. The other parameters values are a = 0.4, b = 1.2, ω = 0.81 and Ω = (√5 + 1) / 2.

4. Conclusion

In the present paper, we numerically studied the effect of NBFM and WBFM signals on chaotic motion in the Brusselator chemical system (Eqs.2 and 3). We demonstrated the effect of the parameters F and G on the dynamics of the system for $\omega = \Omega$ and $\omega \neq \Omega$ cases. With the variation of the amplitudes of the force F and G, the system exhibit period–doubling bifurcation, reverse period–doubling bifurcation, periodic windows, period–bubbles, quasi periodic orbits and chaotic orbits. It is also found that the frequency modulated signal suppresses the critical chaotic behaviour in some parameter ranges. The basic properties of the dynamics of the system are analysed by bifurcation diagram, phase portrait, and Poincare map. The additional features of the system in terms of vibrational resonance, ghost vibrational resonance, parametric resonance etc. deserve for further study.

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